Nuclear matter radii of neutron-deficient Kr isotopes

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Interaction cross sections (σI) of 72, 76, 80Kr on a C target were measured at the projectile fragment separator FRS at GSI. Effective matter radii of the nuclei were deduced by a Glauber model analysis so as to reproduce the experimental σI. An increase of matter radii toward the proton drip-line is observed. We compare the extracted radii to theoretical predictions, which are based on Skyrme-Hartree-Fock-Bogoliubov (SHFB) and on relativistic mean field (RMF) approaches. The enhancement of the matter radius observed in 72Kr is discussed.

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I. INTRODUCTION

Precision measurements of the interaction cross sections (σI) at relativistic energies (≈1 A GeV) allow us to derive nuclear matter radii [1]. Since nuclear matter radii are directly related to the density distributions, measurements of σI are a good tool to search for unusual nuclear structures such as halos and skins. We have performed several experiments and have successfully determined effective rms matter radii (r2m ≡ ⟨r2⟩1/2) of unstable nuclei in the light mass region [2–7]. Such systematic studies have led to the discovery of a new magic number N = 16 in light neutron-rich nuclei [8] and evidence for proton skins in proton-rich Ar isotopes [7]. So far, r2m measured via σI have been determined up to the mass A ~ 40 region.

Recently, an anomalous behavior of r2m of proton-rich Ga, Ge, As, Se, and Br nuclei was reported by measuring the reaction cross sections (σR) at intermediate beam energies [9,10]. The authors of Refs. [9,10] claim to have observed a decrease of r2m with increasing neutron number for all isotopic series they have studied. The nuclear structure of the neutron-deficient Kr isotopes close to the N = Z line, including the self-conjugate nucleus 72Kr, attracts particular interest. These isotopes are located in the middle of the fp shell, around the shell gaps according to the deformed shell model, where competing prolate-spherical or oblate-spherical shape transitions have been predicted for many years. Adding or removing a few nucleons to or from a nucleus can change its nuclear shape dramatically. Recent Coulomb excitation and γ spectroscopy experiments [11–16] have indicated large deformations in this region, and even evidence of shape coexistence for 72, 74, 76Kr.

Gross properties of krypton isotopes such as masses [17,18], half-lives [19], and charge radii, as well as spins and moments [20] have been studied extensively. A measurement of matter radii would provide additional macroscopic information that allows us to determine the ground-state properties of these nuclei. In this paper we present results of r2m of the neutron-deficient isotopes 72, 76, 80Kr studied via their σI measured at relativistic energies, and discuss the systematics of r2m in this region.

II. EXPERIMENT

The experiment was performed at the fragment separator FRS at GSI, Darmstadt [21]. A primary beam of 80Kr accelerated by the SIS synchrotron to an energy of 1.05 A GeV impinged on a Be target at the entrance of FRS to produce secondary beams of 72, 76, 80Kr through projectile fragmentation. A total of three different settings of the FRS were used to select 72, 76, 80Kr as the central beams. A 2512 mg/cm2 thick Be target was used for 72Kr, 1032 mg/cm2 thick for 76Kr, and no production target for 80Kr. A typical primary-beam intensity of 5×108 particles per spill was used to produce 72Kr.

The experimental setup is sketched in Fig. 1. Since the details of the measurements have already been given in Ref. [22], we restrict ourselves here to a short summary of the relevant techniques. At the first focus (F1), we placed a scintillation counter to obtain the start signal for a time-of-flight (TOF) measurement. It also served as an active slit for momentum measurement. It also served as an active slit for momentum definition (∆p/p = ±0.2%). At the momentum-dispersive intermediate focus (F2) of FRS (dispersion: 7 cm/%), we installed another plastic scintillation counter to obtain the TOF stop signal (flight path length l = 17.8 m). The TOF resolution of 33 ps (FWHM) obtained for the primary beam was sufficient to unambiguously identify the mass numbers A of the particles of interest. It should be noted here that the lifetimes of isomeric states, such as τ = 4.53(59) ps for the 2+ state of 72Kr [14] and τ = 41.2(6) ps for the 2+ state of 76Kr [16] are safely shorter than the flight time of ~140 ns between the production target...
The fragment nuclear charge number $Z$ was determined by an energy-loss ($\Delta E$) measurement with the ionization chamber, MUSIC [23]. A $\Delta E$ resolution of 0.6% in $\sigma$ was obtained, corresponding to a $Z$-resolving power of $\Delta Z = 0.33$ (FWHM). Additionally, we positioned two time-projection chambers (TPC) [24] at F2 to tune the separator and to monitor the secondary-beam emittance. The position information at the foils F1 and F2 together with the magnetic field in the second dipole magnet gave $B_0$ of the fragments. Note that the charge state $q$ of the ions can be assumed at this beam energy to be identical to their nuclear charge numbers $Z$ with probability larger than 99.9%, according to GLOBAL [25] calculations. We could thus calculate $A/Z$ and $Z$, providing event-by-event particle identification of the secondary beams impinging at F2.

Secondary beams of $^{72,76}$Kr were delivered to a 2.30 g/cm$^2$ thick carbon reaction target located at F2. The nonuniformity in the thickness of this target was negligible. Particle identification in the second half of FRS, F2–F4, was performed in a manner similar to that used in the first part, F1–F2. To this end, a setup at F4, consisting of two TPCs, an ionization chamber (TEGIC [26]), and a plastic scintillation counter, allowed us to determine $B_0$, TOF, and $\Delta E$.

The principle of the interaction cross section measurements was based on the transmission method. The first and the second half of FRS were used as a spectrometer to identify incoming and outgoing particles, respectively. The question of a possible admixture of the $0^+_2$ shape isomer in $^{72}$Kr will be discussed below in Sec. III.

III. DATA ANALYSIS

Before obtaining the rms matter radii from the measured $\sigma_I$, we estimated the contribution of electromagnetic dissociation (EMD) of the Kr projectiles, $\sigma_{\text{EMD}}$. This cross section can be assumed to be small in view of the small atomic number of the C target; this is verified in a simple calculation shown below. In addition, we paid attention to the admixture of isomeric states in $^{72,76}$Kr.

A. Estimate of the electromagnetic-dissociation contribution

EMD can be assumed to occur via virtual-photon excitation of the giant dipole resonance, followed by particle decay. At the high energy of our experiment, single-step $E1$ excitation is the dominant process. Its strength can be estimated by using the “$E1$ sum rule” [27] of $\sigma = 60 \times N Z/A$ (mb MeV). The integrated flux of virtual photons around a GDR energy of 19 ($\sim 79 \times (A)^{-1/3}$) $\pm$ 2.6 ($\sim 0.5 \times 90 \times (A)^{-2/3}$) MeV is estimated to be 0.036 MeV$^{-1}$ using the equivalent-photon method [28]. Thus $\sigma_{\text{EMD}}$ via this process is estimated to be approximately 38 mb. Significant low-lying $E1$ strength is not expected to occur for the well-bound nuclei of this experiment, therefore an estimate of $\sigma_{\text{EMD}}$ using only the GDR strength is sufficient. Since the EMD contribution of $\approx 38$ mb quoted above is comparable to or smaller than the errors of the interaction cross sections, we use the $\sigma_I$ listed in Table I for further discussion. The uncertainty of 40 mb in $\sigma_I$ corresponds to an uncertainty of 0.08 fm in $\tilde{r}_m$.

B. Population of isomeric states

In the present setup for $\sigma_I$ measurements, the secondary projectile could be produced in an isomeric state that lives long enough to be transmitted to the secondary target at F2. We have therefore checked for this possibility, with the exception of $^{80}$Kr. It is known that there is a low-lying metastable isomeric $0^+_2$ state in $^{76}$Kr [29]. This state is located 346 keV above the $2^+_1$ state. A $0^+_2 \rightarrow 2^+_1$ transition occurs, which is known to be dominant by a factor of 490(19) over the $E0$ transition. Other excited states up to about 4.5 MeV have shorter half-lives than the $0^+_2$ state. The mean lifetime of the $0^+_2$ state is 61(9) ps [29] in the rest frame and 124(18) ps in the laboratory.

![FIG. 1. Experimental setup at the fragment separator FRS.](image-url)

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\sigma_I$</th>
<th>$\tilde{r}_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>2034(21)</td>
<td>4.029(42)</td>
</tr>
<tr>
<td>76</td>
<td>2002(19)</td>
<td>4.002(41)</td>
</tr>
<tr>
<td>72</td>
<td>2195(138)</td>
<td>4.43(27)</td>
</tr>
</tbody>
</table>

TABLE I. Interaction cross sections ($\sigma_I$) in milibarns for Kr isotopes with mass number $A$ and their nuclear effective rms matter radii ($\tilde{r}_m$) in fm.
frame, which is safely much shorter than the flight time of \( \sim 140 \text{ ns} \) between the production target (F0) and the reaction target (F2).

A \( 0^+ \) state occurs also in \(^{72}\text{Kr}\) [12]. Here, the situation should be considered more carefully, since only the \( E0 \) transition \( 0^+_2 \rightarrow 0^+_{g.s.} \) is energetically allowed. The decay by internal conversion is to first order blocked for fully stripped ions. The mean lifetimes of the \( 0^+_{g.s.} \) state in the rest frame and that in the laboratory frame are 38(3) ns [12] and 75(6) ns, respectively. Thus a produced isomeric state may reach the reaction target. At GANIL, the isomeric ratio \( P(0^+_{2})/P(0^+_{g.s.}) \) of the \( 0^+_{2} \) shape isomer in \(^{72}\text{Kr}\) produced by projectile fragmentation of \(^{78}\text{Kr}\) on a \(^{9}\text{Be}\) target was found to be 5.5(12)% at 73 A MeV [30]. Other systematics of isomeric ratios can be found in [31], where isomers (\(^{84}\text{Nb},^{88}\text{Zr},^{90}\text{Mo,}\) and so on) produced via the fragmentation of \(^{78}\text{Kr}\) on a \(^{9}\text{Be}\) target were studied. The population ranges are \( 9(1)\% - 42(11)\% \) for \( J^+ = 5^- \sim 14^+ \).

Since the energy dependence of the production mechanism is unclear, we estimated the isomeric ratio using a fragmentation model based on geometrical abrasion [32]. Under the present experimental conditions, the excitation energy of the prefragments is assumed to be around 27 MeV and the average number of evaporated nucleons to be two. As one can see from Fig. 2, this model predicts the isomeric ratio for \( J = 0 \) to reach 30%. The difference in the charge radius between the ground state and the isomeric state can be found to be 0.005 fm for \(^{79}\text{Kr}\) [20], which amounts to only 0.1%. We arbitrarily assume that the radius of \( 0^+_{g.s.} \) differs from that of \( 0^+_{2} \) by 10% (~0.4 fm). Under these assumptions, the contribution of the \( 0^+_{2} \) state to the measured matter radius is 3%. This is smaller than the uncertainty of 0.27 fm in \( r_m \), and therefore, the mixing of the \( 0^+_{2} \) shape isomer in \(^{72}\text{Kr}\) does not affect the results within the present statistical uncertainty.

![FIG. 2. The angular momentum distribution of \(^{72}\text{Kr}\) via fragmentation of \(^{80}\text{Kr} + ^{9}\text{Be} \rightarrow ^{72}\text{Kr}\) at 1 A GeV as calculated in the geometrical-abrasion model [32]. The distribution is normalized to its maximum value.](image)

C. Determination of matter radii

Based on the Glauber model in the optical limit [33], \( \bar{r}_m \) were deduced from measured \( \sigma_I \). It should be noted that at relativistic beam energies the relation \( \sigma_R = \sigma_I + \sigma_{\text{inelastic}} \) is well approximated by \( \sigma_R \approx \sigma_I [7] \).

A harmonic-oscillator type density distribution, which reproduces the densities of light nuclei well, was used for the target (\(^{12}\text{C}\)). For the projectiles (\(^{72,76,80}\text{Kr}\)), a Fermi type function was employed as the density distribution, \( \rho(r) = \frac{\rho_0}{1 + \exp \left( \frac{r - a}{\frac{1}{2}} \right)} \), where \( R \) and \( a \) denote the half density radius and the diffuseness parameter, respectively. We fixed the diffuseness parameter to be \( a = 0.5 \text{ fm} \) which was obtained experimentally by studying electron scattering from \(^{88}\text{Sr}\) [34]. The resulting \( \bar{r}_m \) of \(^{72,76,80}\text{Kr}\) are listed in Table I and shown in Fig. 3 together with those of \( \text{Ga, Ge, As, Se, and Br isotopes measured at GANIL [9,10]} \).

IV. RESULTS AND DISCUSSIONS

A. Variation of the radius with atomic and neutron number

Recently, Lima et al. [9,10] have measured \( \sigma_R \) of proton-rich \( \text{Ga, Ge, As, Se, and Br isotopes at intermediate energies using a telescope of stacked Si detectors surrounded by Ge detectors. It is observed that for all isotopes} \bar{r}_m \) tends to decrease as the mass number increases, although the experimental uncertainties are relatively large due to the efficiency corrections for the Ge detectors. The present result for the \( \text{Kr isotopes is consistent with this behavior of} \bar{r}_m \). To make the systematic behavior of the radii clear, the atomic number (\( Z \)) dependence of \( \bar{r}_m \) for \( N = 36 \) isotones is plotted in Fig. 4. It is clear that the \( \bar{r}_m \) increase monotonically toward the proton-drip line. The solid and dashed lines indicate RMF predictions with the NL3 and TMA parameter set, respectively. The RMF calculation with the NL-SH interaction presented in Ref. [37] reproduces the systematics of the isotope shifts well, while similar calculations agree with one another but
FIG. 4. Atomic number dependence of $r_m$ for $N = 36$ isotones. Data for Ga, Ge, As, and Se shown by open circles were taken from Ref. [9]. The dotted line shows the $A^{1/3}$ dependence of the matter radii normalized to Ga. The solid [35] and dashed [36] lines are theoretical predictions.

fail to reproduce the atomic number dependence of $r_m$. The experimental atomic number dependence is steeper than what is predicted by theory when adding a proton to a nucleus. The $r_m$ increase more strongly with $Z$ than with $A^{1/3}$.

In Fig. 5 we plot the evolution of $r_m$ and rms proton radii ($\bar{r}_p \equiv (r_{ch}^2)^{1/2}$), which were extracted from charge radii ($\bar{r}_{ch} \equiv (r_{ch}^2)^{1/2}$) [20], using the relation $r_{ch}^2 = r_p^2 - 0.8^2$. The RMF calculation with the NL-SH interaction presented in Ref. [37] reproduces the systematics of the isotope shifts well but once again does not reproduce $r_m$. Recent Dirac-Hartree-Bogoliubov (DHB) calculations [38] also do not show a decrease in $r_m$ with the increase in neutron number for proton-rich isotopes of Ga, Ge, As, Se, and Br, although the DHB calculation describes the total binding energies well. The DHB results show a smooth increase in $r_m$ with both, increasing neutron and increasing proton numbers.

B. Variation of the radius with the deformation

The correlation between the matter radii and deformation for proton-rich Ga, Ge, As, Se, and Br isotopes was studied by the authors of Refs. [9,10]. The excitation energies of the first $2^+$ state for even-even nuclei and those of the first excited state with $J = J_{e.s.} + 2$ for odd-even nuclei were compared with observed $r_m$. They reported that no correlation was seen between an increase in deformation and the increase in radial extension with decreasing $N$ for Ga, Ge, As, Se, and Br.

For krypton isotopes theory predicts a change in the shape of the ground state [39] from predominantly oblate in $^{72}$Kr to prolate in $^{76}$Kr. An oblate ground state ($|\beta_2| = 0.33$) [14] in $^{72}$Kr and a prolate ground state ($|\beta_2| = 0.38$) [15] in $^{76}$Kr have been observed experimentally. It is known that $^{80}$Kr has an oblate ground state deformation ($|\beta_2| = 0.25$) [40].

Figure 6 shows the two-dimensional plot of $\beta_2$ versus ($r_m^2$). Here we followed the procedure described in Ref. [41], which is based on a “pairing-plus-quadrupole” model [42], where the nuclear shapes are parametrized as rotational ellipsoids with the deformation being limited to the quadrupole contribution. Under this assumption, $r_m$ can be written as the parabolic form of $r^2 = (r_{sph}^2) (1 + 0.5 \beta_2^2)$. If we further assume that the radii of the krypton isotopes are simply described by the above two parameters, $\beta_2$ and the spherical part of the nuclear radius ($r_{sph}$), a deviation from the parabolic curve for $^{72}$Kr is seen.

FIG. 5. Neutron number dependence of rms matter and proton radii. Closed symbols indicate the experimental data of $r_m$ (squares) and $\bar{r}_p$ (circles) from $\bar{r}_{ch}$ [20]. Corresponding open symbols connected by lines show theoretical predictions [35].

FIG. 6. Two-dimensional plot of $\beta_2$ versus ($r_m^2$) for $^{72,76,80}$Kr. Open symbols show the experimental data. The curve indicates the result of the fit to the parabolic form of $r_m^2 = (r_{sph}^2) (1 + 0.5 \beta_2^2)$. The closed symbols indicate theoretical predictions by RMF with NL3 (circles) [35], RMF with TMA (squares) [36], Skyrme-Hartree-Fock-Bogoliubov (SHFB) with SKM* (diamonds), and SHFB with SLy4 (triangles). Details of the SHFB calculation are given in Ref. [43]. The corresponding mass number is indicated at each data point.
versus simultaneously reproduce 66Ge is inferred to have a collective oblate shape with moderate 

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In Fig. 6, theoretical predictions are also plotted for comparison. It can be seen that none of the calculations simultaneously reproduce $r_m$ and $\beta_2$. This could be due to a dependence of the shape of the ground state predicted by theory on the effective interactions. The potential-energy curves obtained by these calculations (see, e.g., [36,43]) show two or three local minima close in energy, indicating shape coexistence.

The present work measures $\sigma_I$ without nuclear orientation, thus in principle the deduced $r_m$ are not sensitive to the nuclear shape. The natural interpretation is that the admixture of oblate deformation increases when going to more neutron-deficient nuclei, which more than compensates for the volume effect of decreasing mass.

As shown in Fig. 3, we observe a similar, relatively large increase of $r_m$ in $^{66}$Ge and $^{69}$Se but not in $^{67}$As. The nucleus $^{66}$Ge is inferred to have a collective oblate shape with moderate $\beta_2 \sim -0.23$ and $\gamma \sim -54^\circ$ [44]. The nucleus $^{69}$Se is also interpreted to have a substantial oblate deformation $|\beta_2| \sim 0.24-0.85$ [45], while little evidence for oblate collectivity in $^{67}$As is reported [46]. Therefore, the increase of $r_m$ toward the proton drip-line may be due to other than the axially-symmetric quadrupole deformation such as triaxial or gamma-soft shapes.

C. Variation of the radius with the separation energy

The main driving force for the formation of skin phenomena is the difference between the proton and neutron Fermi energies. The authors of Ref. [7] evaluated the proton skin thicknesses and discussed their correlations with the separation energy differences ($S_p - S_n$) for proton-rich Ar isotopes. The situation in Kr seems to be different, however, from our previous observation in the Ar isotopes. The observed feature of decreasing matter radii with increasing $N$ is difficult to explain supposing a larger neutron than proton radius, and therefore, we find no such correlation between the proton skin thickness and $S_p - S_n$.

In Fig. 7, $r_m$ is plotted against $|S_p - S_n|$. The observed tendency is somewhat similar to that found in Fig. (8b) of Ref. [3]. This fact can be explained by a model in which only the valence nucleons are responsible for the changes in nuclear radii [3].

V. SUMMARY

In summary, we have measured the interaction cross sections of neutron-deficient $^{72,76,80}$Kr isotopes and determined their effective rms matter radii based on a Glauber model analysis in the optical-limit approximation. It was shown that the effective rms matter radii around mass $A \sim 70$ at the proton-rich side increase toward the proton drip-line. The increase of $r_m$ toward the proton drip-line may be due to deformations other than axially-symmetric ones and/or contribution from valence nucleons. Further measurements of $\sigma_R$ of unstable nuclei for the $A > 40$ region are necessary to establish the evolution of $r_m$ on the proton-rich side.

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