Further Study on Coaxial-Probe-Based Two-Thickness-Method for Nondestructive and Broadband Measurement of Complex EM-parameters of Absorbing Material

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SUMMARY Two-Thickness-Method (TTM) based on an open-ended coaxial probe was investigated with an emphasis on uncertainty analysis to perfect this technique. Uncertainty equations in differential forms are established for the simultaneous measurement of complex electromagnetic (EM) parameters in the systematical consideration of various error factors in measurement. Worst-case differential uncertainty equations were defined while the implicit partial derivation techniques were used to find the coefficients in formulation. The relations between the uncertainties and test sample's thicknesses were depicted via 3D figures, while the influence of the coaxial line's dimension on the measurement accuracy is also included based on the same analysis method. The comparisons between the measured errors and theoretical uncertainty prediction are given for several samples, which validate the effectiveness of our analysis.

key words: coaxial probe, two-thickness-method, absorbing material, EM-parameters, permittivity, permeability, uncertainty

1. Introduction

The knowledge of materials’ electromagnetic (EM) parameters, i.e. permittivity ε and permeability μ, is always of necessity in studying varieties of physical phenomena, which involving the interaction between the EM field and matters. Correspondingly, various EM-parameters characterization techniques, such as cavity method, waveguide method and etc., have been proposed. Among them, open-ended coaxial probe measurement technique, which possesses the intrinsic merits such as broadband-capability, opened-structure and wide-compatibility, etc., is essentially applicable for non-destructive and broadband testing of materials’ EM-parameters at microwave frequency [1]–[11]. Tremendous applications and potentials could be found in material science, in medical in-vivo detection and even in large scale integrated circuits industry field, etc. [2].

On the other hand, as well known, to simultaneously measure the both complex EM-parameters, i.e. permittivity ε and permeability μ, via reflection measurement technique, one must obtain at least two reflection coefficients under different test conditions [1]–[3]. Based on the fact that the vector reflection coefficient \( \Gamma(\varepsilon, \mu, f, d, \cdots) \) is a function of the sample’s EM-parameters and sample thickness \( d \), etc., different reflections can be obtained by intentionally measuring the sample with two different thicknesses, i.e. “Two-thickness-method (TTM)” [1]. Since its initialization, this technique was wildly applied in transmission line measurement technique for a long time [1]. Until around the middle of last decade, to satisfy the requirement of non-destructivity and wide-band in measurement, scientists began to combine this method with open-ended coaxial probe technique to simultaneously characterize both complex EM-parameters [2]–[9]. Though it is effective in experimental application, until now, no systematically theoretical uncertainty analysis on coaxial-probe-based TTM has been reported. To perfect this technique, as a complimentary study, the measurement uncertainties of this technique will be analyzed to solve two problems in theory: 1) Before testing, how to appropriately choose the sample’s thicknesses to suppress the uncertainties in measurement; 2) After testing, how to estimate the order of measurement accuracy.

In this paper, an uncertainty analysis for TTM will be performed. Firstly, a brief review on measurement system and the principle of coaxial-probe-based TTM will be given. Then, followed by the formulation of the synthesized uncertainty equation, the uncertainty analysis, supported by 3D figures, will be presented. In addition, the influences of the coaxial line’s size on test uncertainties will also be included based on the same analysis method. The comparisons between the measured errors and theoretical uncertainty prediction are given for several samples, which validate the effectiveness of our analysis.

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2. Two-Thickness-Method (TTM) Based on Open-Ended Coaxial Probe

In TTM, two needed \( \Gamma(\varepsilon, \mu, f, d, \cdots) \) vectors are recorded by measuring a sample with two different thicknesses \( d_1 \) and \( d_2 \), respectively. Then, a right \( \varepsilon \) and \( \mu \) pair should lead the calculated reflection coefficients equal to the measured ones:

\[
\begin{align*}
\Gamma_{m1} &= \Gamma_{c1}(\varepsilon_1, \mu, f, d_1, \cdots) \\
\Gamma_{m2} &= \Gamma_{c2}(\varepsilon_1, \mu, f, d_2, \cdots) \\
\theta_{m1} &= \arg[\Gamma_{c1}(\varepsilon_1, \mu, f, d_1, \cdots)] \\
\theta_{m2} &= \arg[\Gamma_{c2}(\varepsilon_1, \mu, f, d_2, \cdots)] \\
\Rightarrow (1) \\
\end{align*}
\]
where the subscript \textit{m} denotes “measured,” \textit{c} denotes “calculated.” Then, from (1) we are able to work out the both ε\textsubscript{r} = ε\textsubscript{r}\textsuperscript{c} · (1 − \textit{j} tan δ\textsubscript{r}) and μ\textsubscript{r} = μ\textsubscript{r}\textsuperscript{c} · (1 − \textit{j} tan δ\textsubscript{r}), through numerical algorithms, e.g. Newton-Raphson method, etc.

The reflection coefficient against sample’ thickness (APC-7 compatible coaxial probe, the EM-parameters of 9052 is shown in Fig. 6 while that of X1 is shown in Table 1).

![Fig. 1](image)

**Fig. 1** Measurement configuration.

![Fig. 2](image)

**Fig. 2** The reflection coefficient against sample’ thickness (APC-7 compatible coaxial probe, the EM-parameters of 9052 is shown in Fig. 6 while that of X1 is shown in Table 1).

3. Uncertainty Analysis

The degree of accuracy in EM-parameters characterization is limited and lowered because of the errors in the measurement and in modeling. The finite dimension of flange effect [10], an imperfect short-circuited plane and a neglecting of high-order modes in coaxial line [7] lead to modeling errors. Systematic uncertainties of the instrumentation used for reflection coefficients measurement, roughness of sample surface [11], unascertained sample thickness and test frequency corresponds to measurement error. Since most of these error sources and corresponding suppression measures have been separately discussed in some previous literatures [7], [10], [11], we will focus our attention merely on their overall contributions to the uncertainties of measured EM-parameters in this paper. Correspondingly, a synthesized worst-case differential uncertainty analysis for real part of complex permittivity, assuming no cross-correlations and small values in sources of errors, requires that:

\[
\Delta \varepsilon' = \left( \frac{\partial \varepsilon'}{\partial f} \Delta f \right)^2 + \sum_{i=1}^{2} \left[ \left( \frac{\partial \varepsilon'}{\partial \Gamma_i} \Delta \Gamma_i \right)^2 + \left( \frac{\partial \varepsilon'}{\partial \theta_i} \Delta \theta_i \right)^2 + \left( \frac{\partial \varepsilon'}{\partial d_i} \Delta d_i \right)^2 \right]^{1/2}
\]

from which, we can see that the uncertainty is separately evaluated, where \(\Delta[\Gamma_i], \Delta\theta_i\) represents the overall uncertainty of modulus and phase of reflection coefficient due to above-mentioned error sources, respectively. \(\Delta d_i\) represents the uncertainty of the sample thickness, \(\Delta f\) represents that of the test frequency. The subindex \(i\) denotes the measurement under different sample thickness. Similar uncertainty equations also exist for tan δ\textsubscript{r}, \(\mu'\textsubscript{r}\), tan δ\textsubscript{r} respectively.

The remaining work is to find the necessary partial derivatives terms in (3). Correspondingly, implicit differentiation techniques should be used based on the facts that it is reasonable to assume that other magnetic elastomer-type absorbing materials (even other type of absorber), whose attenuation is within 10–30 dB, have the similar reflective characteristics; 3) In region I of Fig. 2, |Γ| declines very fast with the increase of \(d/\lambda\), i.e. a small uncertainty in measured thickness \(d\) will possibly introduce a big uncertainty of |Γ|. So, one must pay great attention to the accuracy of measured sample’s thicknesses if they are selected in region I; 4) Two sample thicknesses cannot be both chosen in area III, because that is contradicted with the TTM’ assumption that reflections must vary with sample’s thickness; 5) Since another key point for sample’s thickness selection is to require the difference of two thicknesses big enough to ensure the degree of independence of two equations (reflections) in (1), we can make a reasonable conclusion that, in TTM, one of the thicknesses is better to be chose at the beginning part of region II of Fig. 2, while the other one at the end part of II or in III. (This point will be further verified by the uncertainty analysis results discussed in next section.)
each EM parameter ($\epsilon'_r$, $\tan \delta_r$, $\mu'_r$ or $\tan \delta_m$) is a function of following independent variables: $[\Gamma]$, $\theta$, $d$, $f$. Firstly, differentiating (3) with respect to $y$, which represents any of the above-mentioned independent variables (or error sources). A linear set of algebraic equations could then be obtained in matrix form:

$$
\begin{align*}
\frac{\partial \Gamma_1}{\partial \epsilon'_r} & \frac{\partial \Gamma_1}{\partial \tan \delta_r} & \frac{\partial \Gamma_1}{\partial \mu'_r} & \frac{\partial \Gamma_1}{\partial \tan \delta_m} & \frac{\partial \Gamma_2}{\partial \epsilon'_r} & \frac{\partial \Gamma_2}{\partial \tan \delta_r} & \frac{\partial \Gamma_2}{\partial \mu'_r} & \frac{\partial \Gamma_2}{\partial \tan \delta_m} \\
\frac{\partial \theta}{\partial \epsilon'_r} & \frac{\partial \theta}{\partial \tan \delta_r} & \frac{\partial \theta}{\partial \mu'_r} & \frac{\partial \theta}{\partial \tan \delta_m} \\
\frac{\partial \mu'_r}{\partial \epsilon'_r} & \frac{\partial \mu'_r}{\partial \tan \delta_r} & \frac{\partial \mu'_r}{\partial \mu'_r} & \frac{\partial \mu'_r}{\partial \tan \delta_m} \\
\frac{\partial \tan \delta_m}{\partial \epsilon'_r} & \frac{\partial \tan \delta_m}{\partial \tan \delta_r} & \frac{\partial \tan \delta_m}{\partial \mu'_r} & \frac{\partial \tan \delta_m}{\partial \tan \delta_m} \\
\frac{\partial \tan \delta_m}{\partial \theta} & \frac{\partial \tan \delta_m}{\partial \epsilon'_r} & \frac{\partial \tan \delta_m}{\partial \tan \delta_r} & \frac{\partial \tan \delta_m}{\partial \mu'_r} & \frac{\partial \tan \delta_m}{\partial \tan \delta_m}
\end{align*}
\right]^{-1}
\begin{align*}
\frac{\partial \Gamma_1}{\partial y} & \frac{\partial \Gamma_2}{\partial y} \\
\frac{\partial \theta}{\partial y} \\
\frac{\partial \mu'_r}{\partial y} \\
\frac{\partial \tan \delta_m}{\partial y}
\end{align*}
\right) =
\frac{\partial \Gamma_1}{\partial \Gamma_2} =
\frac{\partial \Gamma_1}{\partial \Gamma_2} =
\frac{\partial \theta}{\partial \Gamma_2} =
\frac{\partial \mu'_r}{\partial \Gamma_2} =
\frac{\partial \tan \delta_m}{\partial \Gamma_2}
$$

(4)

where all the variables have the same meanings as stated above, while the partial derivation coefficients at both side of equation system could be determined using (1) and (2) with numerical derivation technique. Note that: although all these coefficients are the functions of real EM-parameters, in experiments, we can use the measured (or estimated) ones instead. Then unknowns (bold and italic) in (4), i.e. desired partial derivatives in (3), can be easily solved with any algorithms for linear system.

In the following, we will perform the uncertainty analysis on two typical magnetic elastomer absorbing materials (X1 and 9052) to reach some conclusions. In Fig. 3, the uncertainties of EM-parameters measured with TTM for the absorbing material X1 are depicted as a function of the two sample (electrical) thicknesses at 9.3 GHz respectively. Note that the uncertainty of electrical loss ($\tan \delta_r = 0.02$) is given in absolute form due to its too small value, while the others are in percent form. The uncertainties for all error sources ($\Delta \Gamma_0 = 0.04$, $\Delta h_0 = 2.0\%$, $\Delta d = 0.01$ mm, $\Delta f = 1$ kHz) are obtained based on the analysis in literatures [7], [11], [12] and the author’s experiences. To make best understanding, the mapping contours for the 3D plot were also provided on the bottom of each figure. In these contours, four levels (black, dark gray, light gray and white) of gray-scale, marked in the left-above scale-bars, are used to describe the distributions of the EM-parameters’ uncertainties with the variation of the different thicknesses pairs used in TTM. From these plots, we can conclude that: 1) The symmetrically distributive uncertainties trend to increase along with the decrease of the difference between two measured sample thicknesses, and the extremes $\Delta \rightarrow \infty$ occur as $d_1 \rightarrow d_2$, in which case only one thickness of the sample is measured; 2) The uncertainties distributions for electrical parameters ($\epsilon$) and magnetic one ($\mu$) are not exactly accordant, so when choosing the test sample’s thicknesses, we must make a tradeoff according to required accuracy for the different parameters; 3) For X1, the given measurement accuracy requirements ($\Delta \epsilon'_r/\epsilon'_r \leq 10\%$, $\Delta \tan \delta_r \leq 0.2$, $\Delta \mu'_r/\mu'_r \leq 20\%$, $\Delta \tan \delta_m/\tan \delta_m \leq 35\%$) demand a sample-thickness selection range as $0.1\lambda_g < d_1(d_2) < 0.2\lambda_g$, $d_2(d_1) > d_1(d_2) + 0.25\lambda_g$ ($\lambda_g$ is the wavelength in sample), which is accordant with the conclusions from Fig. 2 discussed in Sect. 2. In terms of the author’s experience, though not exact, this range could be also used as a reference in se-
lecting the thickness pairs, in TTM, for elastomer-type (even other type) absorbers, whose attenuations are in the range of 10–30 dB/λg.

For further confirmation, in Fig. 4, the uncertainties for another elastomer-type absorbing material 9052 at 5 GHz, 9 GHz, 14 GHz are illustrated respectively. Then we can know: 1) For 9052, although not exactly the same, the trend of uncertainties distributions against the test sample’s electrical thicknesses are similar at three frequencies; 2) Compared with Fig. 3, we can know, the trend of uncertainties distributions of 9052 is also similar with those of X1; 3) For 9052, if we choose two samples’ thicknesses \(d_1, d_2\) satisfying \(0.1\lambda_g < d_1(d_2) < 0.2\lambda_g, \ d_2(d_1) > d_1(d_2) + 0.25\lambda_g\), the measurement accuracy will be better than \(\Delta|\varepsilon'|/\varepsilon' \leq 10\%, \ \Delta|\tan \delta\varepsilon| \leq 0.15, \ \Delta|\mu'\varepsilon'|/\mu' \leq 20\%, \ \Delta|\tan \delta\mu|/\tan \delta\mu \leq 20\%\), which verified the above-summarized sample-thickness selection condition.

Note that: The author doesn’t mean that: to select the samples’ thicknesses within the range above will lead the best accuracy. Strictly speaking, to optimize the samples’ thickness must be based on Eq. (3). The proposal of the above empirical rule is just for the readers, who don’t want to perform so complicated uncertainty analysis procedure, to be able to make a comparatively reasonable selection of samples’ thicknesses in TTM.

In Fig. 5, the influences of coaxial line’s dimension on the EM-parameters’ uncertainties are pictured using 2D contours based on sample X1 (\(d_1 = 1.3\) mm, \(d_2 = 2.56\) mm).

For intuitive observation, the impedance of coaxial line is also shown in the same figure by a set of straight lines. Some conclusions can be made as: 1) Given a certain impedance, the measurement uncertainties decrease as the size of coaxial line increases; 2) To obtain a higher test accuracy (especially for magnetic parameters), the coaxial line impedance should be chosen within 35 Ω and 75 Ω; 3) When choosing the dimension of coaxial line, the cutoff frequency of high order mode must be considered. (In Fig. 5, the white color depicts the region where the cutoff frequency of the first high-order mode TE11 is less than 9.3 GHz); 4) In practical experiments, another necessarily-considered factor for the coaxial probe is to match the currently available measurement instruments. As a consequence, some coaxial probe with standard dimension, e.g. APC-7 compatible size, is always selected to avoid such problems as calibration, mismatching etc.

4. Experimental Confirmation

An experimental measurement system, the same as that used in [3], has been set up. An ANA (Agilent8722ES) is used as the reflectometer while an APC-7 compatible open-ended coaxial probe was fabricated to measure the short-circuited absorbing coatings as depicted in Fig. 1. Accordingly, calibration can be easily completed using standard calibration kits before measurement. In Table 1, the typical absorbing material X1 under different thickness pairs was measured on
an ANA (Agilent8722ES), while the corresponding uncertainties, determined by (3), are also given in absolute form \(\Delta|\varepsilon'|, \Delta|\tan\delta|, \Delta|\mu'|, \Delta|\tan\delta_{\mu}|\) to make an intuitive comparison between the measurement errors \(\varepsilon = |\text{measured value} - \text{reference value}|\) and our uncertainties prediction. We see that TVM determines the EM-parameters with high accuracy when using the first two thickness pairs, while the other two pairs give the results with comparatively low accuracy, which agrees well with the tendency of uncertainty prediction shown in Fig. 3.

Broadband frequency-swept measurements were carried out on another absorbing material 9052 under two thicknesses \((d_1 = 1.22 \text{ mm}, d_2 = 2.44 \text{ mm})\), which are depicted in Fig. 6 in the form of electrical thickness against frequency. Both the measured EM parameters and reference data (provided by Marconi Company) are shown in Fig. 6. In addition, to make an intuitive comparison, the estimated absolute uncertainties based on (3) are also denoted in this figure by “I” line. We can see from Figs. 6 and 7 that: 1) The measured EM parameters have an acceptable agreement.
with the reference data, which verified the effectiveness of TTM. 2) At most of the frequency points, our uncertainty equations make a good estimation on the up-limit of the actual errors; 3) At a few points, however, the actual errors are bigger than our estimations. One possible reason for this is because of the inaccuracy of the measured data at these frequency points. The second one is due to the errors in estimation of above-indicated $\Delta |f_{0}|$, $\Delta \theta$, $\Delta d$, $\Delta f$, which will sometimes deteriorate the feasibility of these uncertainty formulas.

5. Conclusions

An uncertainty analysis of “Two-Thickness Method” (TTM) has been performed for the simultaneous measurement of complex EM-parameters $\varepsilon$ and $\mu$ using an open-ended coaxial probe, in the systematic consideration of all the error sources associated with computation, modeling, measurements, etc. Implicit partial derivation technique has been used, based on the TTM, to provide the relations between the uncertainties and sample thickness pair in 3D figures. An empirical thickness-selection rule of TTM was obtained as $0.1 \lambda_{g} < d_{1}(d_{2}) < 0.2 \lambda_{g}$, $d_{2}(d_{1}) > d_{1}(d_{2}) + 0.25 \lambda_{g}$, for the measurement of the magnetic elastomer-type absorbing materials. Meanwhile, the influence of the coaxial probe’s dimension on the EM-parameters’ measurement accuracy was also included. Both the single-frequency measurement under different sample thickness couples and the broadband frequency-swept testing were conducted on the typical absorbing materials, which validated the feasibilities of our analysis.

References


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