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Monoenergetic Neutrino Beam for Long Baseline Experiments

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In an electron capture process by a nucleus, emitted neutrinos are monoenergetic. We study a long baseline experiment with such a completely monoenergetic neutrino beam. This talk is based on \textsuperscript{1,2}

1. Introduction

Numerous observations on neutrinos from the sun\textsuperscript{3}, the atmosphere\textsuperscript{4,5}, reactors\textsuperscript{6}, and the accelerator\textsuperscript{7} suggest that neutrinos are massive and hence there is mixing in the lepton sector.

Within the three generation framework, two of the mixing angles and the two mass squared differences are well determined.\textsuperscript{8} To determine these parameters much more precisely and to observe effects from the other two mixing parameters, $\theta_{13}$ and the CP phase $\delta$, there were several ideas proposed for next generation neutrino oscillation experiments.\textsuperscript{9,10,11,12}

For a precision measurement, it is obviously better to have an experiment using neutrinos with controllable and precisely known energy. To achieve this we consider making use of a nucleus which absorbs an electron and emits a neutrino:

$$ (Z, A) + e^- \rightarrow (Z - 1, A) + \nu_e, $$

where $Z$ is the electric charge of the mother nucleus and $A$ is its mass number. In this case neutrinos have a line spectrum and their energy is precisely known. Therefore by accelerating the mother nuclei appropriately with the Lorenz boost factor $\gamma_m$, we can control the neutrino energy and make use of monoenergetic neutrinos in an oscillation experiment.

The experimental setup is very simple. We need an accumulating ring as usual\textsuperscript{11} to circulate the nuclei. This ring is equipped with an electron injected at the entrance of the decay section which has length $X$ and an apparatus for separation of nuclei and electrons at the end of the decay section. The energy of injected electrons must be tuned precisely so that their boost factor $\gamma_e$ is the same as that of the nuclei $\gamma_m$, $\gamma_e = \gamma_m$. The separation section at the end must be constructed
such that it can separate the nuclei and electrons properly in order to circulate the nuclei until they capture an electron. It may be implemented by photon injection and a strong magnet.

The range of neutrino energies, $E_{\nu}$, in the laboratory frame is given by

$$0 < E_{\nu} < 2\gamma_m Q$$

where $Q$ is the energy difference between the mother and the daughter nuclei and $Q \ll M$. The range of $E_{\nu}$ that can be made use of is the important point of this paper.

It is most probable that the beam is focused in the direction of a detector. In the center of the beam direction $E_{\nu} = 2\gamma_m Q$. Therefore the appropriate boost factor $\gamma_m$ for the experiment is derived from the baseline length $L$ and the relevant mass square difference $\delta m^2$:

$$\frac{\delta m^2 L}{4E_{\nu}} \bigg|_{E_{\nu}=2\gamma_m Q} = P,$$

where $P$ is the desired oscillation phase at the maximum neutrino energy which is determined by the physics goal. For example, if one wants to observe the oscillation at the first maximum, then $P = \pi/2$. From eq.(3),

$$\gamma_m = \frac{\delta m^2 L}{8P} \frac{1}{Q}.$$  

Since in the rest frame of the mother nuclei, the distance between the decay section and the detector is $L' \equiv L/\gamma_m$, larger $\gamma_m$ means higher neutrino flux at the detector. It scales proportionally to $\gamma_m^2$. We observe from eq.(4) that a lower $Q$ value is better. However a lower $Q$ means, in general, a larger half-life $\tau$. The mother nuclei should capture an electron frequently enough, otherwise we cannot get a neutrino beam of a sufficient strength. This means

$$\tau \gamma_m < T$$

where $T$ is an appropriate time interval within which we require that all the mother nuclei should experience the process (1). Therefore, since in this kind of experiments data are taken for several years, $T$ is of the order of a month or at most a year. This requires that $\gamma_m$ should be smaller which conflicts with the requirement of getting a higher-flux neutrino beam mentioned below eq. (4). To satisfy both the requirements, we have to find a nucleus which has a smaller $Q$ value and a shorter half-life $\tau$. In the following $\gamma_m \gg 1$ and nucleus mass $M \gg Q$ are used in our derivations.

Let us now examine the theoretical aspects of this idea in more detail.

\section{Case (i) Small $Q$ value and High $\gamma_m$: Neutrinos only from electron capture}

As one of the first candidates we study here $^{110}_{50}$Sn. Theoretically this gives the best example for our scenario. Its half-life $\tau_{Sn}$ is 4.11 hour. Its $J^P$ is $0^+$. It decays
into the excited state of $^{110}$In, with $1^+$ whose energy level is 343 keV. Since the mass difference is 638 keV, the energy difference between neutral $^{110}$Sn and $^{110}$In, $\Delta_{Sn}$, is 295 keV, that is, the energy of the emitted neutrino is 295 keV minus to the binding energy. For example, since the K shell binding energy, $E^K_{In}$ of $^{110}$In is 28 keV, the emitted neutrino energy in the rest frame of Sn, $Q_{Sn} = \Delta_{Sn} - E^K_{In}$ is 267 keV.\(^b\)

Then the appropriate acceleration of $^{110}$Sn is

$$\gamma_{Sn} = 378 \left( \frac{\delta m^2}{2.5 \times 10^{-3} \text{eV}^2} \right) \left( \frac{L}{100 \text{km}} \right) \left( \frac{\pi/2}{P} \right). \quad (6)$$

In the rest frame of $^{110}$Sn, the distance $L'_{Sn}$ is given by

$$L'_{Sn} = 264 \left( \frac{2.5 \times 10^{-3} \text{eV}^2}{\delta m^2} \right) \left( \frac{P}{\pi/2} \right). \quad (7)$$

Therefore if the "fiducial" detector radius is larger than 264 m, half of the neutrinos will hit the detector. Because of the reason mentioned below the theoretically most interesting oscillation phase is $P = \pi/3$ and hence 264 m is realistic. This size of a detector is not unrealistic. Incidentally, since $\gamma_{Sn} = 567$, $\gamma_{Sn} \tau_{Sn} = 96$ days, satisfying eq. (5). This efficiency should be compared with the case of a neutrino factory or a beta beam. In a neutrino factory\(^10\) the distance, $L''_\beta$ corresponding to $L'_{Sn}$ is $O(10)$ km and hence even if the area of the detector perpendicular to the neutrino beam is of $O(100) m^2$, only 0.01% of the neutrinos are used. Similarly in a beta beam experiment $L'_\beta$ is $O(1)$ km and only 1% of neutrinos are used. Therefore, even if we have an amount of $^{110}$Sn which is 2 orders of magnitude smaller than the number of nuclei in a beta beam experiment, say $^8$He, we will have the same physics reach. That is, the "quality factor"\(^11\) is much better. The quality factor is given by the inverse of $L'$. Furthermore, since the neutrino energy is much more clearly determined in this experiment, we have better precision experiment.

There is another interesting feature for sufficiently high $\gamma_m$. As we have seen, almost all neutrinos pass through the detector. Therefore we have a wide range of neutrino energies and by measuring the interaction point the neutrino energy can be "measured" precisely. The energy of a neutrino, which is detected at a distance $R$ from the center of the beam, is easily calculated (in the large $\gamma_m$ limit):

$$E_\nu(R) = \frac{2\gamma_m Q}{1 + R^2/L''_\beta^2}. \quad (8)$$

\(^a\)Our picture for K shell electron capture is that a neutral mother captures its K shell electron and bears a neutral daughter with one K shell hall and one electron in outer orbit. Therefore, exactly speaking, we need to take into account the binding energy of an electron in the outer orbit, $E_o$ which will fall into the K shell finally. This raise the neutrino energy by amount of $E_o$, though we will omit this here.

\(^b\)Since an electron is captured not only from K shell but also other orbits, there are several lines depending on from which shell an electron is captured. It should be included to consider the detail.
Table 1. Candidate Nuclei for case (i). $\gamma_m$ is determined by $P = \pi/3$ for a detector at $L = 100\text{km}$ and $\delta m^2 = 2.5 \times 10^{-3}\text{eV}^2$ using eq.(4). The energy unit is keV. $\text{N}[E]$ means the excited state of the nucleus $\text{N}$ with energy $E[\text{keV}]$. Also the energy difference between mother and daughter nuclei is given in unit keV. The unit for the lifetime $\tau$ (rest frame) and $\tau\gamma_m$ (lab. frame) is given by $\text{m(minute)}, \text{h(hour)}, \text{d(day)}$. “Detector Size” indicates the radius within which a half of the emitted neutrinos are contained at the detector distance, see eq.(9).

The neutrino energy range is determined by eq.(8),

$$\frac{2\gamma_m Q}{1 + R_{max}^2/L^2} < E_\nu < 2\gamma_m Q,$$

(9)

where $R_{max}$ is the “fiducial” detector diameter. For example, if $D = L'$, then half of the emitted neutrinos hit the detector and their energy range is $\gamma_m Q \leq E_\nu \leq 2\gamma_m Q$. The range of the oscillation phase varies from $\pi/3$ to $2\pi/3$, from which we can explore the oscillation shape around the oscillation maximum very precisely.

For the position resolution $\delta R(\delta R^2 = 2R\delta R)$, the energy resolution is given by

$$|\delta E_\nu| = \frac{2\gamma_m Q R^2 / L^2}{(1 + R^2/L^2)^2} \Rightarrow \left| \frac{\delta E_\nu}{E_\nu} \right| = \frac{\delta R^2 / L^2}{(1 + R^2/L^2)^2}. $$

(10)

In the rest frame of the mother nucleus, monoenergetic neutrinos are emitted isotropically. In a solid angle $d\Omega$ in the rest frame, the number of neutrinos is distributed uniformly. The solid angle $d\Omega = 2\pi \sin \theta d\theta$ corresponds to

$$2\pi \sin \theta d\theta = \frac{4\pi}{(1 + R^2/L^2)^2} \frac{dR^2}{L^2}$$

(11)

and in terms of the neutrino energy

$$d\Omega = 2\pi \sin \theta d\theta = \frac{2\pi}{\gamma_m Q} \frac{dE_\nu}{E_\nu}. $$(12)

Thus we have a neutrino beam uniformly distributed in its energy. As a detector can measure the energy and the interaction point, by combining these two measurement, we can determine the neutrino energy very precisely. This specific feature in a beta-capture beam arises from the fact that neutrinos are monoenergetic in the rest frame of the mother nucleus.

In Table 1, we list candidate nuclei for this case (i).

3. Sensitivity for case (i)

Here we show the sensitivity to the oscillation parameters $\theta_{13}$ and $\delta_{CP}$. 
3.1. Setup

We assume a water cherenkov detector with a fiducial mass of 500 kt. The large detector mass allows to collect enough statistics that is needed to gain from the superb energy resolution and can have large geometrical size in order to have a enough broad energy window, since the minimal measurable energy depends on the maximal distance from the beam center. We assume the geometry of the detector to be as shown in Figure 1. The radius of the detector is set to $R_{\text{max}} = 100$ m so that the depth is still approximately 64 m and a reconstruction of the cherenkov rings and electron/muon identification remains possible. The position measurement should be optimized for this kind of experimental setup and reach at least a resolution of 30 cm, which has been the estimated vertex resolution at Super-K for fully-contained single ring events. Furthermore, the vertex resolution for muon events, i.e. the monobeam signal events, is slightly better than for electron events and can even reach a resolution of 25 cm in the energy window of interest.

For having neutrino energies beyond the cherenkov threshold and allow for electron/muon discrimination, we only discuss monobeam setups with neutrino energies above 400 MeV. Above 400 MeV a signal efficiency is approximately 0.55 for the appearance measurement of muon neutrinos. We assume the background rejection to be at a level of $10^{-4}$. We assume a systematical uncertainty of 2.5% for the signal events and 5% for the background events. The energy window of the analysis is given by eq.9, so the energy window is completely fixed after the baseline $L$ and...
the acceleration factor of the ions $\gamma_m$ is chosen. So finding an optimal Setup is more complicated as it is for example in the case of beta beams, since choosing a perfect pair of $L$ and $\gamma$ to exactly measure at the first oscillation maximum can suffer from an energy window that is too small to allow resolving correlations and degeneracies. However, adjusting the baseline to smaller baselines in order to have a lower minimal energy also shifts the oscillation maximum to lower energies, while going to higher values of $\gamma$ not only shifts the maximal energy but also the minimal energy to higher values. So, the whole energy window moves away from the oscillation maximum although it is broadened. Therefore, in the next sections we discuss the potential and performance of the following different reference scenarios of monobeam setups:

- **Setup I**: The water cherenkov detector with a fiducial mass of 500 kt is located at a baseline of $L=600$ km, the mother nuclei $^{110}_{50}$Sn are accelerated with $\gamma = 2500$ and 10 years of data taking are assumed at the number of $10^{18}$ electron capture decays per year.

- **Setup II**: The water cherenkov detector with a fiducial mass of 500 kt is located at a baseline of $L=250$ km, the mother nuclei $^{110}_{50}$Sn are accelerated with $\gamma = 2000$ and 10 years of data taking are assumed at the number of $10^{18}$ electron capture decays per year.

- **Setup III**: The water cherenkov detector with a fiducial mass of 500 kt is located at a baseline of $L=600$ km, the mother nuclei $^{110}_{50}$Sn are accelerated with $\gamma = 900$ and $\gamma = 2500$ consecutively, and 5 years of data taking are assumed in each of the two phases so that as for Setup I and II the total running time is 10 years. The number of $10^{18}$ electron capture decays per year is assumed for both phases.

Setup I is located at the first oscillation maximum, but the energy window is not very broad compared to the width of the oscillation maximum peak, therefore we also discuss the second scenario, Setup II, with a broader energy window which on the other hand is located slightly off the first oscillation maximum at higher neutrino energies due to the smaller baseline. Then again, because of the smaller baseline higher event rates can be obtained at Setup II. With Setup III we discuss the potential in resolving the correlations and degeneracies with a monobeam experiment by a combination of data from the first oscillation maximum and also the second oscillation maximum. This combination should be a powerful tool to resolve the degeneracies and the importance of the second oscillation maximum has been discussed in \cite{16}. Since the first oscillation maximum phase at Setup III is comparable to Setup I, the gain from the additional measurement at the second oscillation maximum can directly be read off the comparison of Setup I and Setup III. The exact width of the corresponding energy windows of the setups and their location respectively to the oscillation maxima are shown in Figure 2. The appearance probability $P(\nu_e \rightarrow \nu_\mu)$ is plotted for $\sin^2 2\theta_{13} = 0.01$ and three choices of $\delta_{CP}$ (the other oscillation parameters are chosen as in Eq. (13)). The yellow (grey) bands
indicate the energy window of the analysis for Setup I and III in the left-hand side and Setup II in the right-hand side. It can be seen that the energy window for the choice of $L=600\text{km}$ and $\gamma = 900$ is essentially only a very narrow band while for the higher values of $\gamma$ indeed a broader energy window can be covered over the whole radius of the detector. However, the energy window of Setup I is too narrow to cover the first oscillation maximum for the different choices of $\delta_{\text{CP}}$. For $\delta_{\text{CP}} = 0$ the peak of the first oscillation maximum lies inside the energy window of the analysis but for the maximally CP violating values for $\delta_{\text{CP}}$ the peak moves outside the energy window. The energy window of Setup II lies above the first oscillation maximum independent of $\delta_{\text{CP}}$ but we will show in the next subsections that Setup I will suffer more from correlations and degeneracies than Setup II since the latter benefits from a higher event rate due to the smaller baseline and the larger energy window, where the superb energy resolution can evolve.

For reasons of comparison and to put the performance of the monobeam setups into perspective we will compare the results to a standard neutrino factory setup with a 50 kt MID detector at a baseline of $L=3000\text{km}$ and a parent energy of the stored muons $E_{\mu} = 50\text{GeV}$. This neutrino factory Setup Is taken from (NuFact-II scenario) with $1.06 \cdot 10^{21}$ useful muon decays per year, five years in each polarity (corresponding to $5.3 \cdot 10^{20}$ useful muon decays per year and polarity for a simultaneous operation with both polarities), so that the total running time is 10 years as for the monobeam setups.

The analysis throughout this work is performed with the GLoBES software\textsuperscript{18}
and the incorporated poissonian $\chi^2$-analysis. Since the monobeam only measures $\nu_\mu$-appearance and could additionally only observe $\nu_e$-disappearance, the leading atmospheric parameters $\sin^2 2\theta_{23}$ and $|\Delta m^2_{31}|$ cannot be determined as would be the case at a neutrino factory with a measurement in the $\nu_\mu$-disappearance channel. Thus, correlations with the leading atmospheric parameters would spoil the potential of the monobeam experiment alone, as also would be the case for a beta beam for the same reasons. Therefore, we adopt the same technique as in [19] and add the $\nu_\mu$-disappearance information from a simulation of the superbeam experiment T2K. The corresponding appearance information is excluded, so that information on $\sin^2 2\theta_{13}$ and $\Delta_{\text{CP}}$ is solely collected by the monobeam experiment (see [19] for details).

As input or so-called true values within the simulations, we use, unless stated otherwise the following parameter values, close to the current best fit values:

\[
\begin{align*}
\Delta m^2_{31} &= 2.5 \cdot 10^{-3} \text{ eV}^2 \quad \sin^2 2\theta_{23} = 1.0, \\
\Delta m^2_{21} &= 8.2 \cdot 10^{-5} \text{ eV}^2 \quad \sin^2 2\theta_{12} = 0.83.
\end{align*}
\] (13)

3.2. Sensitivity to $\sin^2 2\theta_{13}$

The sensitivity to $\sin^2 2\theta_{13}$ is calculated under the hypothesis of true $\sin^2 2\theta_{13} = 0$. The sensitivity limit at a certain confidence level is then the maximal fit value of $\sin^2 2\theta_{13}$ that still fits the simulated data at the chosen confidence level, i.e. it would be the lower bound to $\sin^2 2\theta_{13}$ that the experiment could achieve in case of vanishing true $\sin^2 2\theta_{13}$. It is well known, that the main problem is to resolve the correlations with the other oscillation parameters and the so-called eight-fold degeneracy. In Figure 3 the sensitivity to $\sin^2 2\theta_{13}$ is shown at the $3\sigma$ confidence level as a function of the number of decays per year for the monobeam scenarios at $L=600\text{km}/\gamma = 2500$, $L=250\text{km}/\gamma = 2000$, and $L=600\text{km}/\gamma = 900 + \gamma = 2500$. The vertical lines indicate the reference setups at a number of $10^{18}$ ion decays per year. In each plot the lowest curve represents the pure statistical limit to $\theta$ and the colored bands show how the sensitivity degrades if also systematics (blue/dark grey band), correlations (green/middle grey band), and degeneracies (yellow/bright grey band) are taken into account. The final achievable sensitivity limit to $\sin^2 2\theta_{13}$ is given by the upper curve. Obviously the statistical and systematical sensitivity limit to $\sin^2 2\theta_{13}$ at all three scenarios in Figure 3 can reach to very small values of $\sin^2 2\theta_{13}$ due to the very large statistics in the water cherenkov detector. However, the monobeam scenario at $L=600\text{km}/\gamma = 2500$ can resolve the correlations not until an exposure of $10^{17}$ decays per year. The point where the degeneracies can be resolved is reached not until approximately $10^{20}$ decays per year, which of course is beyond any feasibility. So despite the improvement of the statistical limit with higher exposures the final sensitivity limit to $\sin^2 2\theta_{13}$ stays relatively stable a approximately $\sin^2 2\theta_{13} \approx 10^{-2}$ independent of the number of decays per year. The monobeam scenario at a baseline of $L=250\text{km}$ and $\gamma = 2000$ suffers from the same problem. First, the sensitivity limit does only slightly improve and almost stays
Fig. 3. The sensitivity to $\sin^2 2\theta_{13}$ at the 3$\sigma$ confidence level for the monobeam scenarios $L=600\text{km}/\gamma = 2500$, $L=250\text{km}/\gamma = 2000$, and $L=600\text{km}/\gamma = 900 + \gamma = 2500$ as a function of the number of decaying ions per year including statistics, systematics, correlations, and degeneracies. The lowest curve represents the pure statistical sensitivity limit to $\sin^2 2\theta_{13}$ and the colored bands indicate the effect of switching on systematics (blue/dark grey), correlations (green/middle grey), and degeneracies (yellow/bright grey) so that the final sensitivity limit is given by the upper curve.

stable. Beyond exposures of $10^{18}$ decays per year this scenario starts to resolve the degeneracies and the sensitivity limit to $\sin^2 2\theta_{13}$ improves significantly. From Figure 3 it becomes clear, that the technique of a high gamma monobeam with its superb energy resolution in a narrow energy window is not able to resolve the correlations and degeneracies in a measurement at just one $\gamma$. The scenario at a baseline of $L=600\text{km}$ allows to measure in the second oscillation maximum since for $L=600\text{km}$ this maximum is located above the cherenkov threshold and events can be collected. The lower plot of Figure 3 shows the sensitivity limit to $\sin^2 2\theta_{13}$ for such a scenario, where 5 years data taking at $\gamma = 900$ and 5 years data taking at $\gamma = 2500$ is combined. Now, the correlations and degeneracies can be already resolved for lower exposures. We checked that it is not necessary to split up the two data taking phases into an equal period of five years each. The ability to resolve the
correlations and degeneracies still remains if only 2 years data taking at $\gamma = 900$ are combined with 8 years at $\gamma = 2500$ and the final sensitivity would be even slightly better since then more statistics could be collected at the first oscillation maximum.

For reasons of comparison, the sensitivity to $\sin^2 2\theta_{13}$ at Setup I, Setup II, and Setup III are again shown in the left-hand side of Figure 4 and confronted with the sensitivity limit obtainable at the standard neutrino factory scenario. The neutrino factory also suffers from the correlations and degeneracies. But as can be seen in the right-hand side of Figure 4 the difference is that the neutrino factory can almost resolve the degenerate solution. There, the projected $\Delta \chi^2$ is shown as a function of the fit value of $\sin^2 2\theta_{13}$ for the degenerate solution with the wrong sign, i.e. inverted hierarchy while the positive $\Delta m^2_{31}$ was taken as input true value. The degenerate solution appears for the neutrino factory scenario at a $\Delta \chi^2$ only slightly below the $3\sigma$, while the degenerate solution for Setup I appears at $\Delta \chi^2 = 0$ and thus fits as good as $\sin^2 2\theta_{13} = 0$. On the other hand, with Setup III there does not appear a second local minimum in the projected $\Delta \chi^2$ so the combination of first and second oscillation maximum data gives a strong tool to resolve the degeneracy. However, resolving the degeneracies remains the main problem if one want to reach to very small values of $\sin^2 2\theta_{13}$ and one could also think of a combination of a monobeam setups with the anti-neutrino running of a standard beta beam scenario.
3.3. Sensitivity to CP violation

Due to the continuous intrinsic $\sin^2 2\theta_{13}$-$\delta_{\text{CP}}$-degeneracy a total rates analysis of appearance data of neutrinos only would give continuous bands as allowed regions in the $\sin^2 2\theta_{13}$-$\delta_{\text{CP}}$ plane. If combined with a second band from appearance data...
of anti-neutrinos only two intersections, the true and the degenerate allowed region remain. Adding the spectral information obtained with conventional energy resolution, the degenerate solution can be resolved in most cases. This is the planned procedure at superbeam experiments, neutrino factories as well as beta beam experiments to resolve the $\sin^2 2\theta_{13}$-$\delta_{\text{CP}}$-degeneracy. However, at a monobeam experiment only neutrino appearance is observable and the question arises, if and under which circumstances the superb energy resolution abilities of a monobeam could in principle compete in resolving the $\sin^2 2\theta_{13}$-$\delta_{\text{CP}}$-degeneracy. Since we found in the last section that the ability in resolving the degeneracies does not appear until a large number of decays per year, we will fix this value to $10^{18}$ decays per year in all the following considerations and only discuss the fixed scenarios Setup I, Setup II, and Setup II. In Figure 5 the allowed regions in the $\sin^2 2\theta_{13}$-$\delta_{\text{CP}}$-plane at the 3$\sigma$ confidence level are shown for different choices of input true values. This figure is for illustrative purposes only and no correlations with the other oscillation parameters is considered, i.e. they are kept fixed to the values of Eq. (13). The left column is for Setup I ($L=600\,\text{km}/\gamma = 2500$), the middle column is for the Setup II ($L=250\,\text{km}/\gamma = 2000$), and the right column shows the allowed regions obtained for the standard neutrino factory setup for reasons of comparison. The bands, indicated by the solid grey lines, represent the corresponding allowed regions at the 3$\sigma$ confidence level if only total rates are considered. As expected, the total rates allowed regions for the monobeam scenarios are bands that do not restrict $\delta_{\text{CP}}$ at all whereas for the neutrino factory already the parameter space of $\delta_{\text{CP}}$ is restricted due to the information from neutrino and anti-neutrino data. If spectral information is included to the analysis, the neutrino factory allowed regions are not influenced significantly and only the small degenerate solutions can be excluded, but for the monobeam scenarios because of the superb energy resolution wide parts of the bands can be excluded and only smaller allowed regions remain that are comparable in size to the allowed regions from the neutrino factory scenario. However, in some cases of choices of true values still degenerate solutions remain. As mentioned before, we have ignored correlations with the other oscillation parameters and also the sign($m_{31}$)-degeneracy here. In all of the further considerations, we will focus on the sensitivity to CP violation if also these correlations and all degeneracies are taken into account.

The sensitivity to any CP violation is shown in Figure 6 for Setup I (upper left-hand side plot), Setup II (upper right-hand side plot), Setup II (lower left-hand side plot), and the neutrino factory scenario (lower right-hand side plot) at the 1, 2, 3, 4, and 5 $\sigma$ confidence level from bright grey/yellow (1$\sigma$) to red/dark grey (5$\sigma$). Sensitivity to any CP violation is given for a pair of true values $\sin^2 2\theta_{13}$-$\delta_{\text{CP}}$ if the CP conserving values $\delta_{\text{CP}} = 0$ and $\delta_{\text{CP}} = \pi$ do not fit the simulated reference data if all correlations and degeneracies are taken into account. It is known, that the standard neutrino factory suffers from the sign($\Delta m^2_{31}$)-degeneracy in some areas of the parameter space ($\sin^2 2\theta_{13} \approx 10^{-2.5}$ and $\delta_{\text{CP}} \approx -\pi/2$), because of the so-called “$\pi$-transit”, i.e. the degenerate solution fitted with wrong sign of $\Delta m^2_{31}$ contains the
Fig. 6. Sensitivity to any CP violation at 1 (yellow/bright grey), 2, 3, 4, and 5σ (red/dark grey) after 10 years of data taking as a function of the true values of sin²2θ_{13} and δ_{CP}. The sensitivities are shown for the monobeam scenarios Setup I (upper left-hand side plot), Setup I (upper right-hand side plot), Setup III (lower left-hand side plot) and a standard neutrino factory (lower right-hand side plot) for reasons of comparison. For a pair of true values within the shaded regions the CP conserving fit values δ_{CP} = 0 and δ_{CP} = π can be excluded at the respective confidence level.

CP conserving value for δ_{CP} = π (see [17] for details). As can be seen from Figure 6, Setup I suffers strongly from correlations and degeneracies at larger true values of sin²2θ_{13} whereas Setup II performs better. Within the interval δ_{CP} ∈ [−π, 0] Setup II does not suffer from any correlations and degeneracies anymore and gives better results than the neutrino factory in the same interval. In the interval δ_{CP} ∈ [0, π] Setup II and the neutrino factory perform in a comparable manner, only for larger true values of sin²2θ_{13} ≥ 10^{-2} the neutrino factory looses sensitivity to CP violation for values of δ_{CP} near the CP conserving values. This effect is due to the uncertainty of the matter density along the baseline which strongly affects the performance of a neutrino factory at large values of sin²2θ_{13} because of the very long baseline. The best sensitivity to any CP violation is found for Setup III. Here, the combination of data from the first and second oscillation maximum can resolve the degeneracies that appear at the baseline of L=600km for Setup I. Additionally the sensitivity
to CP violation of Setup III reaches to significant smaller values of $\sin^2 2\theta_{13}$ at the maximally CP violating values $\delta_{\text{CP}} = \pm \pi/2$. We checked that, as also was the case for sensitivity to $\sin^2 2\theta_{13}$, a combination of 2 years at $\gamma = 900$ and 8 years at $\gamma = 2500$ would also already allow to give this performance. The results from Figure 6 are finally summarized in Figure 7. The fraction of $\delta_{\text{CP}}$ parameter space where sensitivity to any CP violation is given at the 3\(\sigma\) confidence level is shown as a function of true $\sin^2 2\theta_{13}$ for the considered scenarios. Note, that a CP fraction of 1 can never be achieved, since values near the CP conserving values can never be distinguished due to finite statistics.

4. \textit{case (ii) Monoenergetic neutrino and Continuous energy neutrino:}

Next we consider the nucleus $^{48}\text{Cr}$. It decays into an excited state of $^{48}\text{V}$ whose energy level is 420keV. The mass difference is 1659keV and $\Delta_{\text{Cr}}$ is 1239 KeV. The half-life is 21.56 hours.\(^{13}\) The K shell binding energy, $E_{\text{K}}^V$, of the daughter nucleus $^{48}\text{V}$ is 5.465 keV\(^{14}\). Since $Q_{\text{Cr}} = \Delta_{\text{Cr}} - E_{\text{K}}^V$ is larger than $2m_e$, twice of the electron mass, it can not only capture an electron but also emit a positron:

$$^{48}\text{Cr} + e^- \rightarrow ^{48}\text{V} + \nu_e \text{ & } ^{48}\text{Cr} \rightarrow ^{48}\text{V} + e^+ + \nu_e.$$ \(^{(14)}\)
Table 2. Candidate Nuclei for case (ii). $\gamma_m$ is determined by $P = \pi/2$ instead $\pi/3$. For $^{113}_{50}$Sn, the lifetime is adjusted by its branching ratio to EC, 8.9%. The last column, the branching ratio for the electron capture and the positron emission, is calculated by using eq.(15) and eq.(16).

Assuming that there are 2 K shell electrons in the mother nucleus $^{48}_{24}$Cr, the rate for the capture process, $\Gamma_c$, is proportional to \(^{20}\)

$$\Gamma_c \propto 2\pi \left\{ (Q_{Cr}/m_e) \right\}^2 (\alpha Z)^3 = 0.196.$$  \hspace{1cm} (15)

Here $m_e$ is the electron mass. The rate for positron emission, $\Gamma_{e^+}$, is proportional to \(^{20}\)

$$\Gamma_{e^+} \propto \int_{1}^{w_0} x \sqrt{x^2 - \frac{1}{1}} (w_0 - x)^2 F(x, Z) dx = 0.004,$$  \hspace{1cm} (16)

$$F(x, Z) = 2(1 + \gamma) \left\{ 2\pi r \right\}^{2\gamma-3} \exp(-\pi \nu) \left[ \frac{\Gamma(\gamma - 2\nu)}{\Gamma(2\gamma + 1)} \right]^2. \hspace{1cm} (17)$$

Here $F(x, Z)$ is the Fermi function \(\gamma \equiv (1 - \alpha Z)^{1/2}, \nu \equiv \alpha Z x/p, p = \sqrt{x^2 - 1}, \alpha \) the fine structure constant =1/137 and $r$ the radius for a nucleus in units of $m_e^{-1}$ \(^{c}\) and $w_0 = (\Delta_{Cr} - m_e)/m_e$ is the maximum positron energy in units of the electron mass. Thus the electron capture process is dominant (98.0%) and hence a neutrino beam with well-controlled energy is available.

In table 2 we list other examples of nuclei which have even lower $Q$ and shorter $\tau_{13}$. $^{18}_{9}$F dominantly decays by positron emission while for $^{48}_{24}$Cr and $^{113}_{50}$Sn the electron capture process dominates.

Since $Q_{Cr}$ is higher than in the previous case, the appropriate $\gamma_{Cr}$ is lower and hence the quality factor is worse than in the previous case. Therefore, we need to store much more $^{48}_{24}$Cr nuclei than $^{110}_{50}$Sn:

$$\gamma_{Cr} = 82 \left( \frac{\delta m^2}{2.50 \times 10^{-3} eV^2} \right) \left( \frac{L}{100 km} \right) \left( \frac{\pi/2}{P} \right), \hspace{1cm} (18)$$

which means that the neutrinos at the detector are completely monoenergetic as can be seen from eq.(9). There is essentially no position dependence of neutrino energy at the detector.

Therefore we cannot explore the energy dependence of the oscillation without changing the beam energy as previously discussed. However, this problem may be

\(^{c}\)For numerical calculations we take $r = 10^{-3}$. However the numerical results here does not depend on $r$ within a few % accuracy.
solved by the use of continuous neutrino associated with positron emission. We can control the boost factor $\gamma_m$ very well and hence the highest neutrino energy at a detector is completely determined by it.\textsuperscript{d} This allows a very accurate calibration of neutrino energy. Furthermore, the energy of the line spectrum and that of the continuous one are clearly separated and simultaneous observation of two distinct energy region gives a useful information on the Unitarity triangle\textsuperscript{22}. Thus having a line and a continuous spectrum simultaneously, we may get better oscillation parameter reach.

5. Conclusion and Discussion

We have studied how the neutrino energy in oscillation experiments can be controlled better than with other ideas that are currently discussed. By electron capture, a nucleus emits a monoenergetic neutrino. Therefore by accelerating the mother nuclei, we can get a well-controlled neutrino beam. To achieve 100% electron capture rate, we need to use a nucleus with a low $Q$ value, lower than $2m_e$. In general, such a nucleus has a long half-life. Furthermore, since we accelerate it with very large boost factor $\gamma_m$, it becomes almost stable. Though this conflicts with the fact that the nucleus must decay within a sufficiently short time interval (see eq.(5)), there are several candidates listed in table 2. With these nuclei, we can control the neutrino energy. Since $\gamma_m$ is very large, a neutrino beam is so well concentrated in the forward direction that almost all neutrinos can be used for oscillation experiments. This significantly reduces the required number of mother nuclei.

As a result of such a high $\gamma_m$, in principle, we don’t need to measure the neutrino energy at the detector since by measuring the detected position we can calculate its energy and hence we have another way of observing the energy dependence of the oscillation.

Theoretically there are only advantages, but these nuclei are so heavy that it is very energy consuming to accelerate them to the ideal $\gamma_m$. Also it may be hard to get enough nuclei even if the required number of nuclei is significantly small. As a compromise, we have also studied nuclei with a higher $Q$ value. These nuclei not only capture an electron but also emit a positron. From the latter process neutrinos with a continuous spectrum are emitted. Furthermore as $Q$ is higher, $\gamma_m$ must be smaller. These facts spoil some of the good features mentioned above. However since we have neutrinos with a line spectrum and a continuous spectrum simultaneously, we may get another good feature for this kind of beams. study.

In this kind of a beta-capture beam, we can produce only a $\nu_e$ beam. To study CP violation we need a $\bar{\nu}_e$\textsuperscript{23} or $\nu_\mu$\textsuperscript{16} beam. On the contrary to $e^-$ capture case, since $e^+$ cannot be bound by nuclei, it is almost impossible to have a sufficiently strong $\bar{\nu}_e$ beam. Instead we can make use of $\mu$ capture to get a monoenergetic $\nu_\mu$ beam, though, since the mass of $\mu$ is very high, the emitted neutrinos have a very

\textsuperscript{d}Similar idea is found in \textsuperscript{21} too.
high energy. We must find a nucleus whose daughter has a mass higher than that of the mother by $O(\mu \text{ mass})$ so that the energy of $\nu_\mu$ in the rest frame of the mother nucleus is sufficiently low.

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